ECE317 : Feedback and Control

Lecture :
Time response - Introduction

Dr. Richard Tymerski
Dept. of Electrical and Computer Engineering
Portland State University

Course roadmap

Modeling

- ✓ Laplace transform
- ✓ Transfer function
- ✓ Block Diagram Linearization
- ✓ Models for systems
  - ✓ electrical
  - ✓ mechanical
  - example system

Analysis

Stability
- Pole locations
- Routh-Hurwitz

Time response
- Transient
- Steady state (error)

Frequency response
- Bode plot

Design

Design specs

Frequency domain

Bode plot

Compensation

Design examples

Matlab & PECS simulations & laboratories

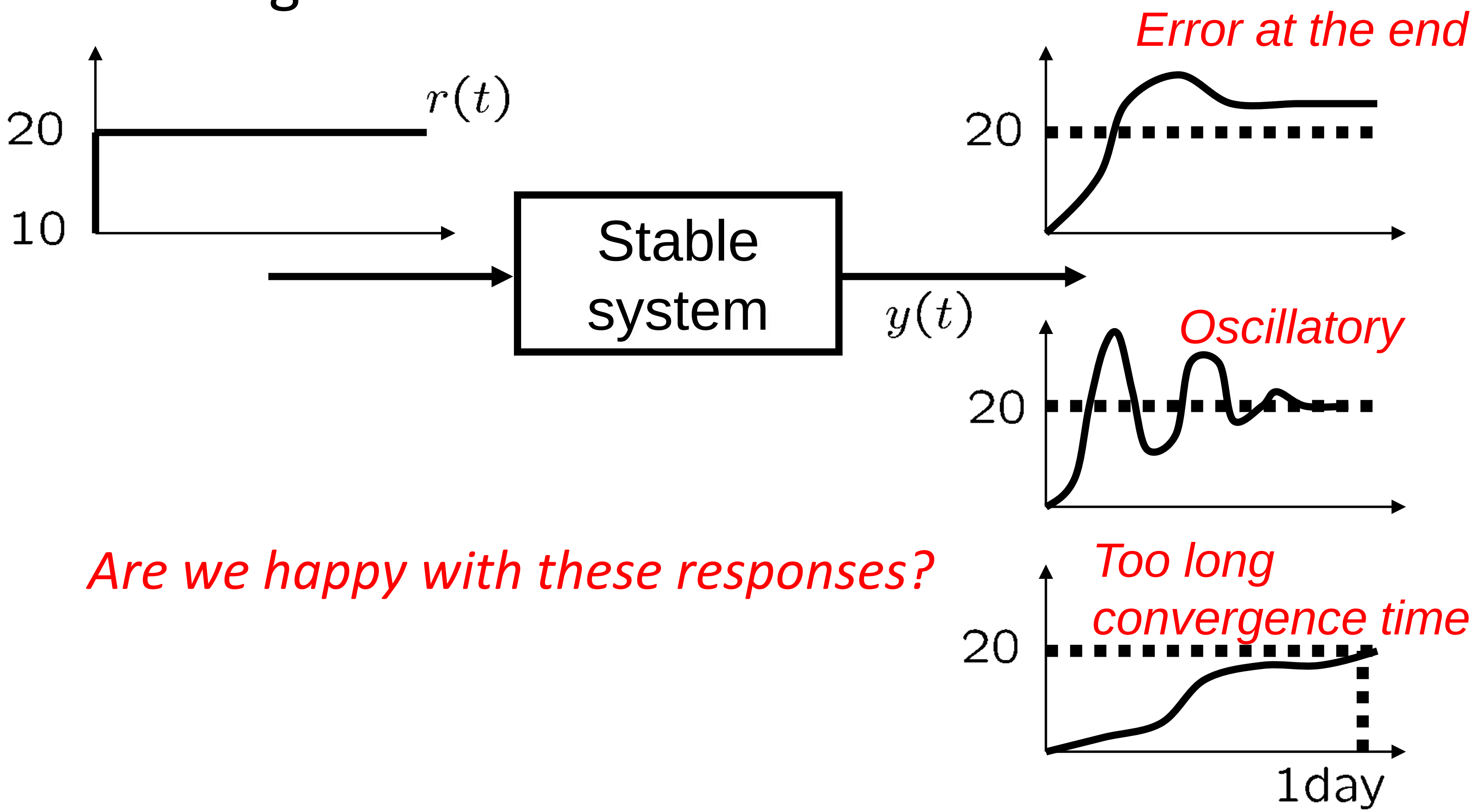
What's to come …

- We will subsequently learn about system stability and how to assess and achieve it.
- Stability is a necessary requirement, but not sufficient for most control systems. (next slide)
- Specifications other than stability
  - How to evaluate a system quantitatively in t-domain?
  - How to give design specifications in t-domain?
  - What are the corresponding conditions in s-domain?

    (we will design controllers in the s-domain).

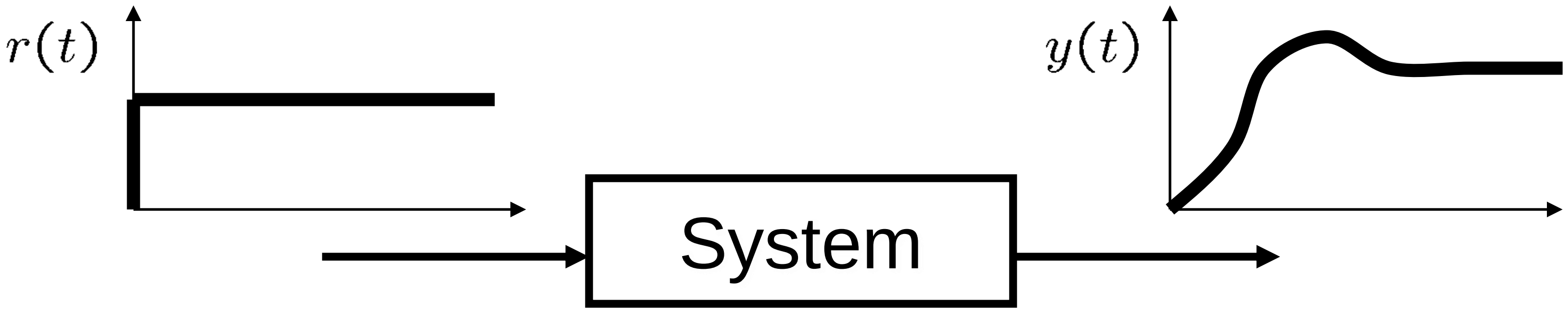
Temperature control example

- We want to change the room temperature from 10 to 20 degree.



Are we happy with these responses?

Time response



- We would like to analyze (stable) system's property by applying a *test input* *r(t)* and observing a time response *y(t)*.
- Time response is divided as

$$y(t) = y_t(t) + y_{ss}(t)$$

Transient (natural) response

$$\lim_{t\to\infty} y_t(t) = 0$$

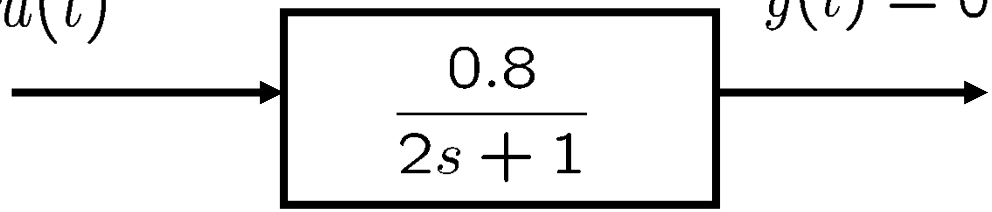
Steady-state (forced) response

(after y_t dies out)

Ex: Transient & steady-state responses

$$r(t) = u(t)$$

$$y(t) = 0.8 - 0.8e^{-\frac{t}{2}}$$

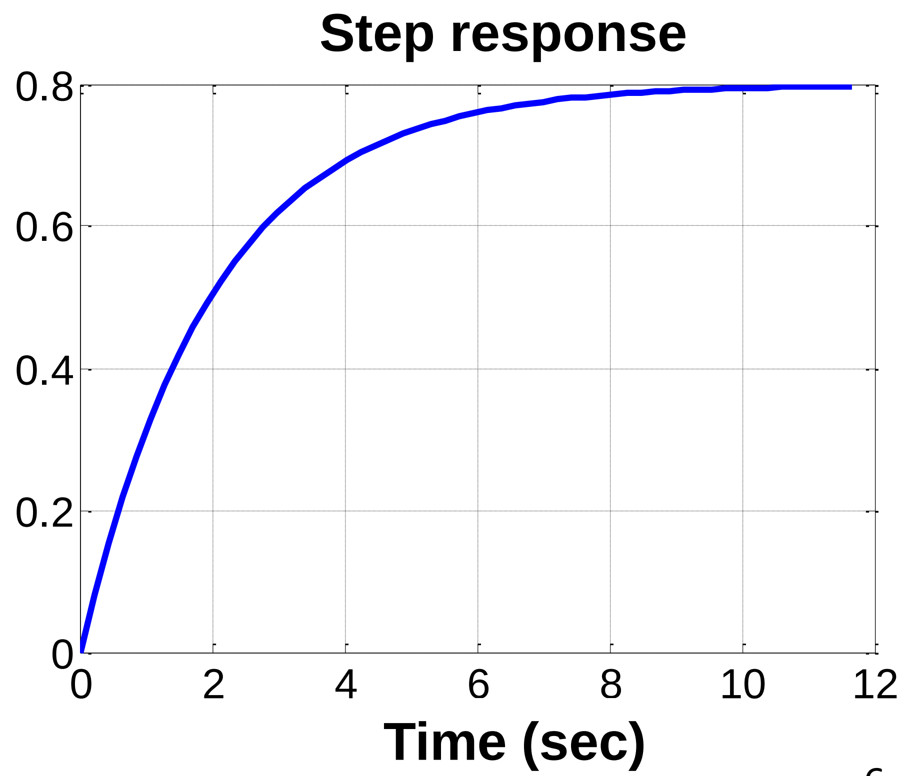


- Transient response

$$y_t(t) = -0.8e^{-\frac{t}{2}}$$

- Steady-state resp.

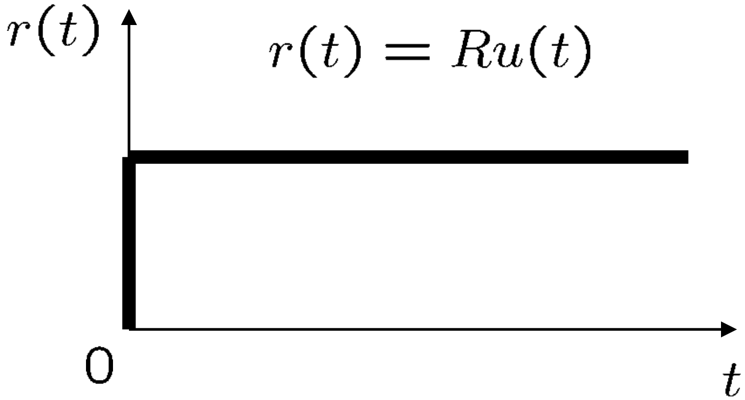
$$y_{ss}(t) = 0.8$$



Usage of time responses

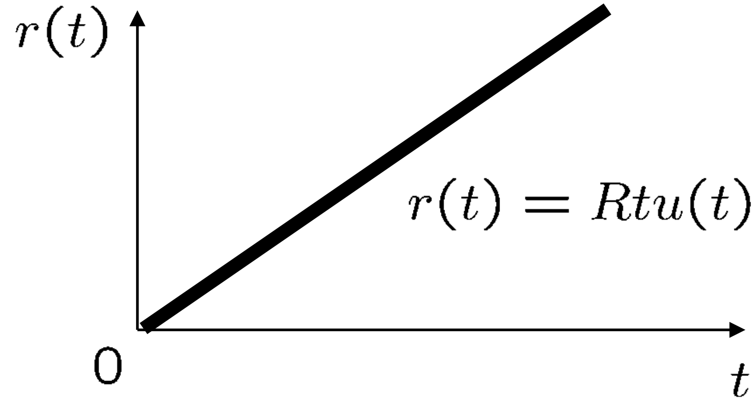
- Modeling
  - Some parameters in the system may be estimated by time responses.
- Analysis
  - A system can be evaluated by seeing transient and steady-state responses. (Satisfactory or not?)
- Design
  - Given design specs in terms of transient and steady-state responses, controllers are designed to satisfy all the design specs.

Typical test inputs



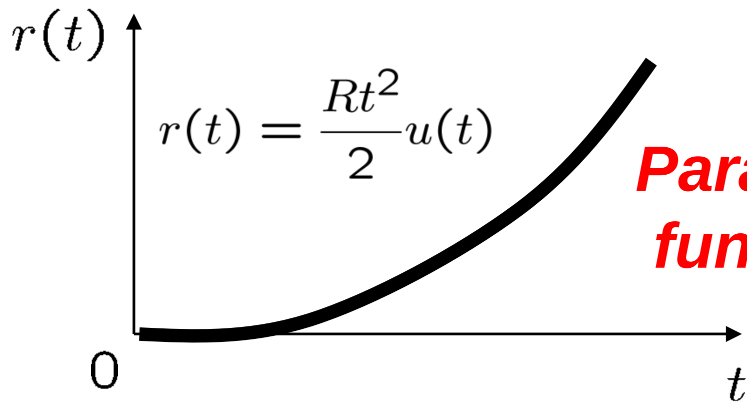
Step function

(Most popular)



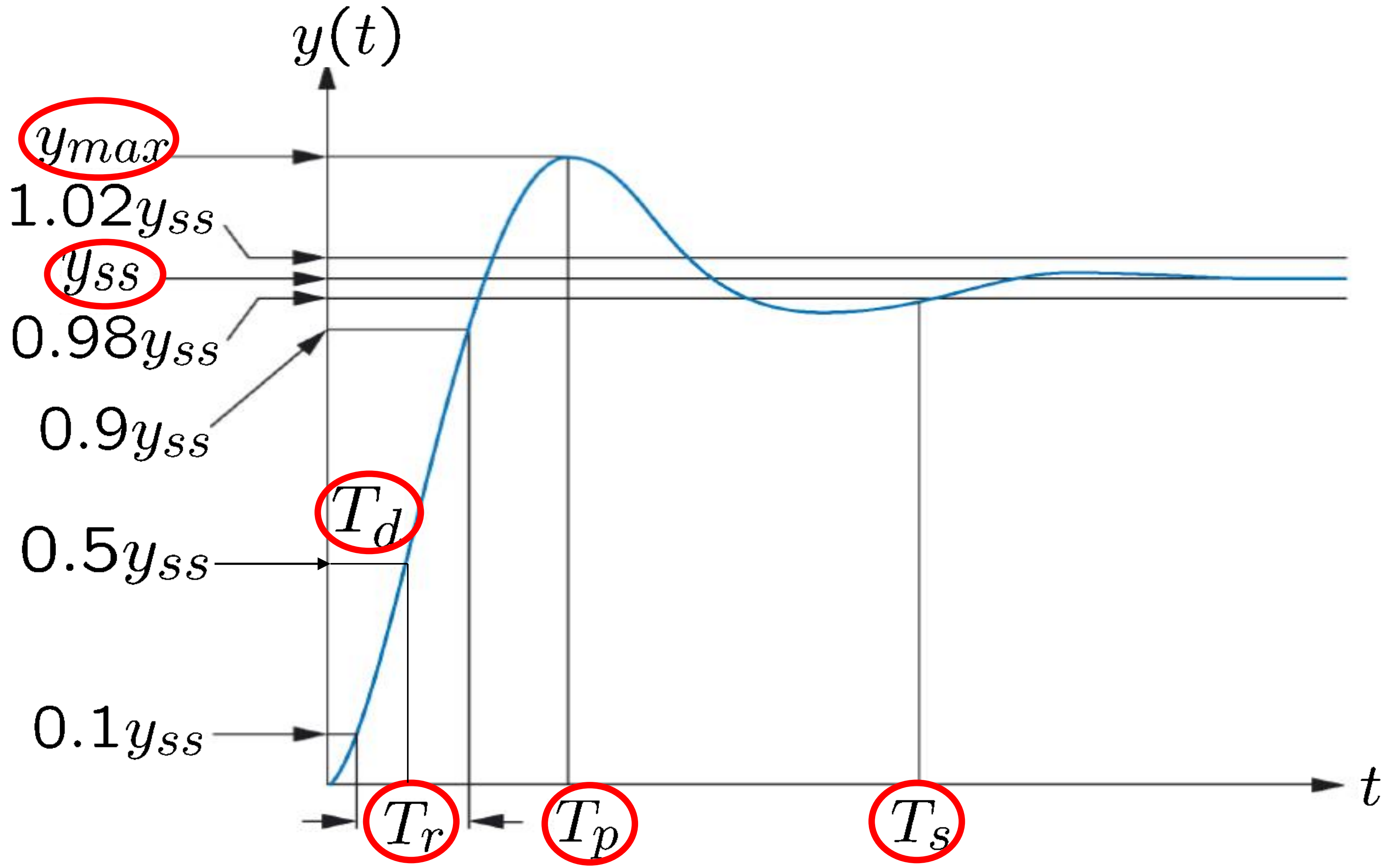
Ramp function

***Sinusoidal input* will be dealt with later.**

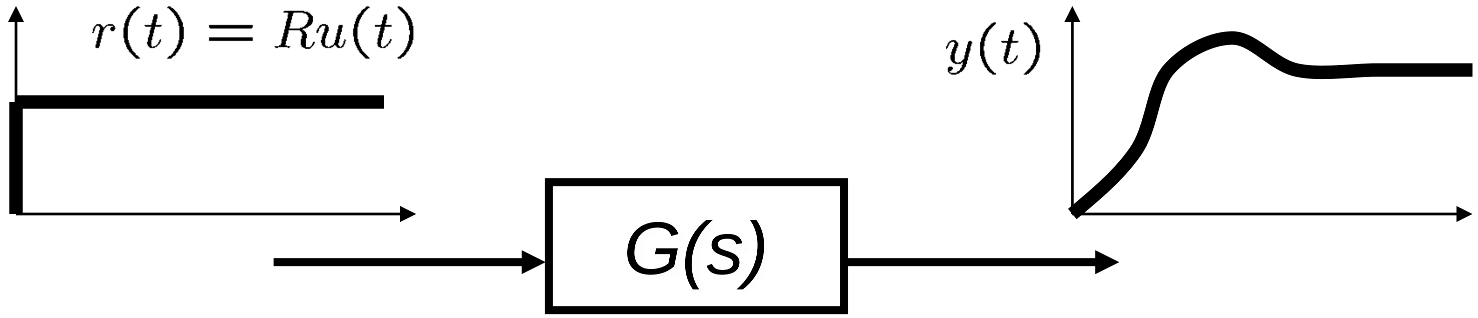


Parabolic function

Typical step response



Steady-state value for step input

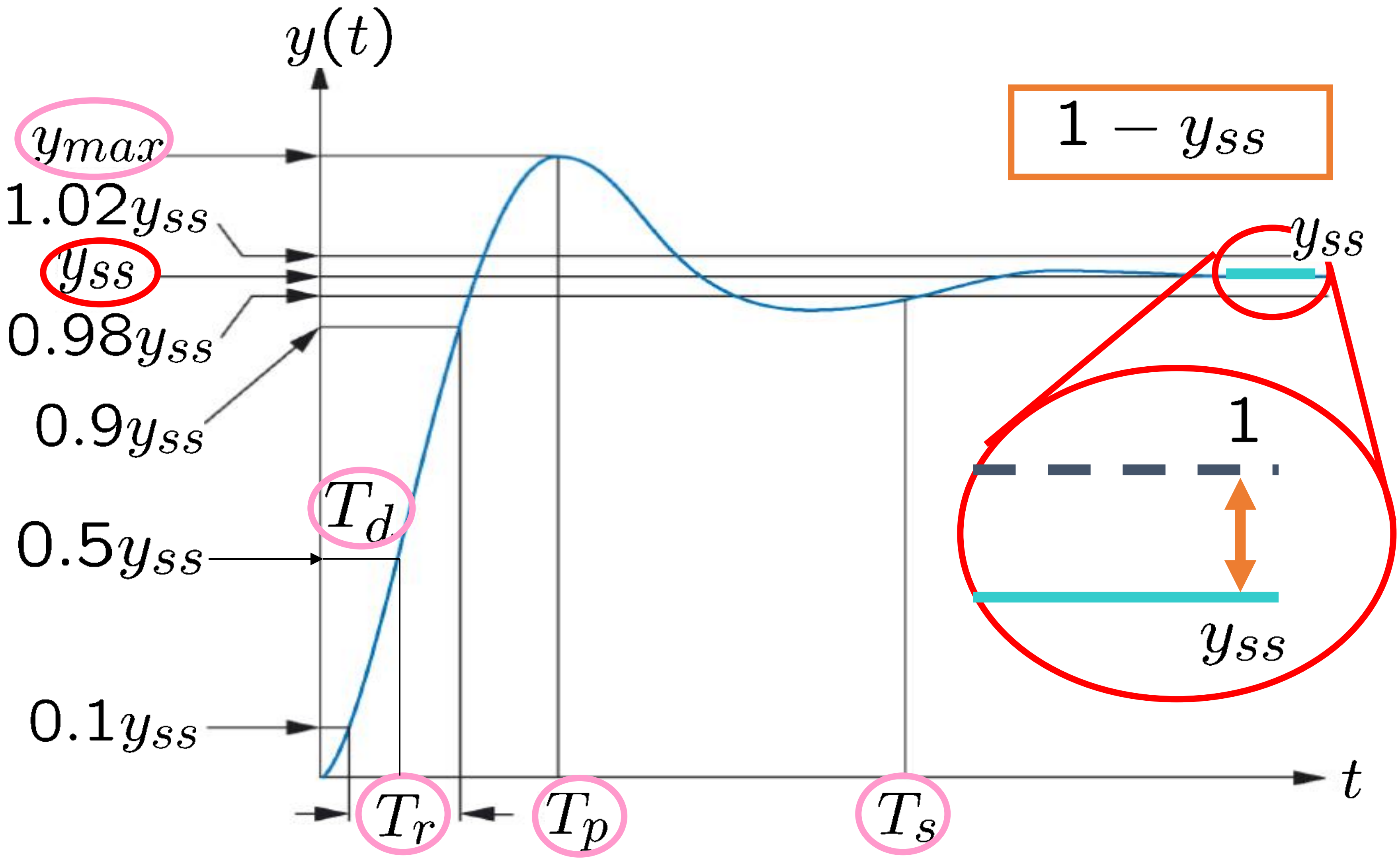


- Suppose that G(s) is stable.
- By the final value theorem:

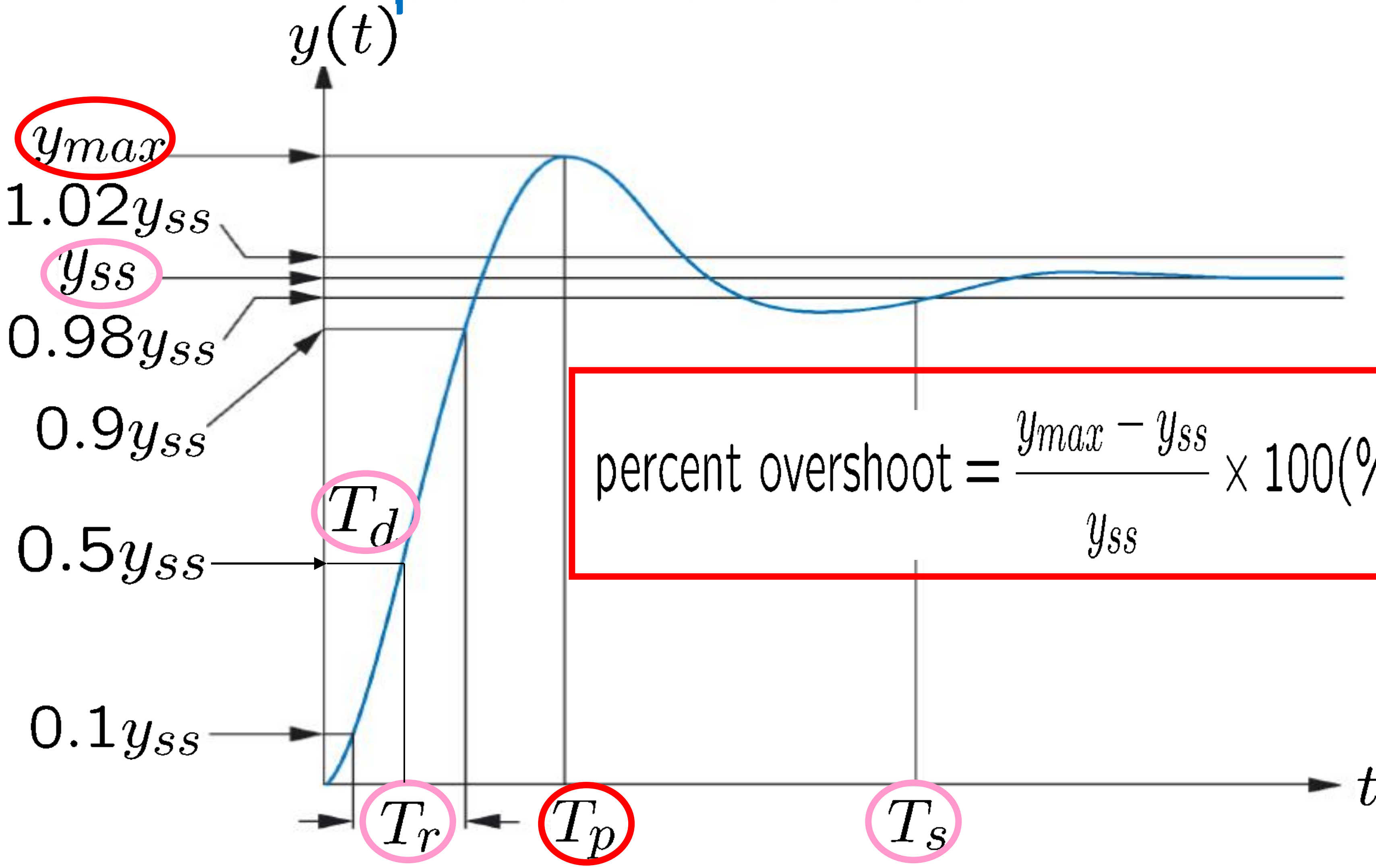
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sG(s)\frac{R}{s} = RG(0)$$

- Step response converges to some finite value, called *steady-state value* y_{ss}

Steady-state error for input *u(t)*

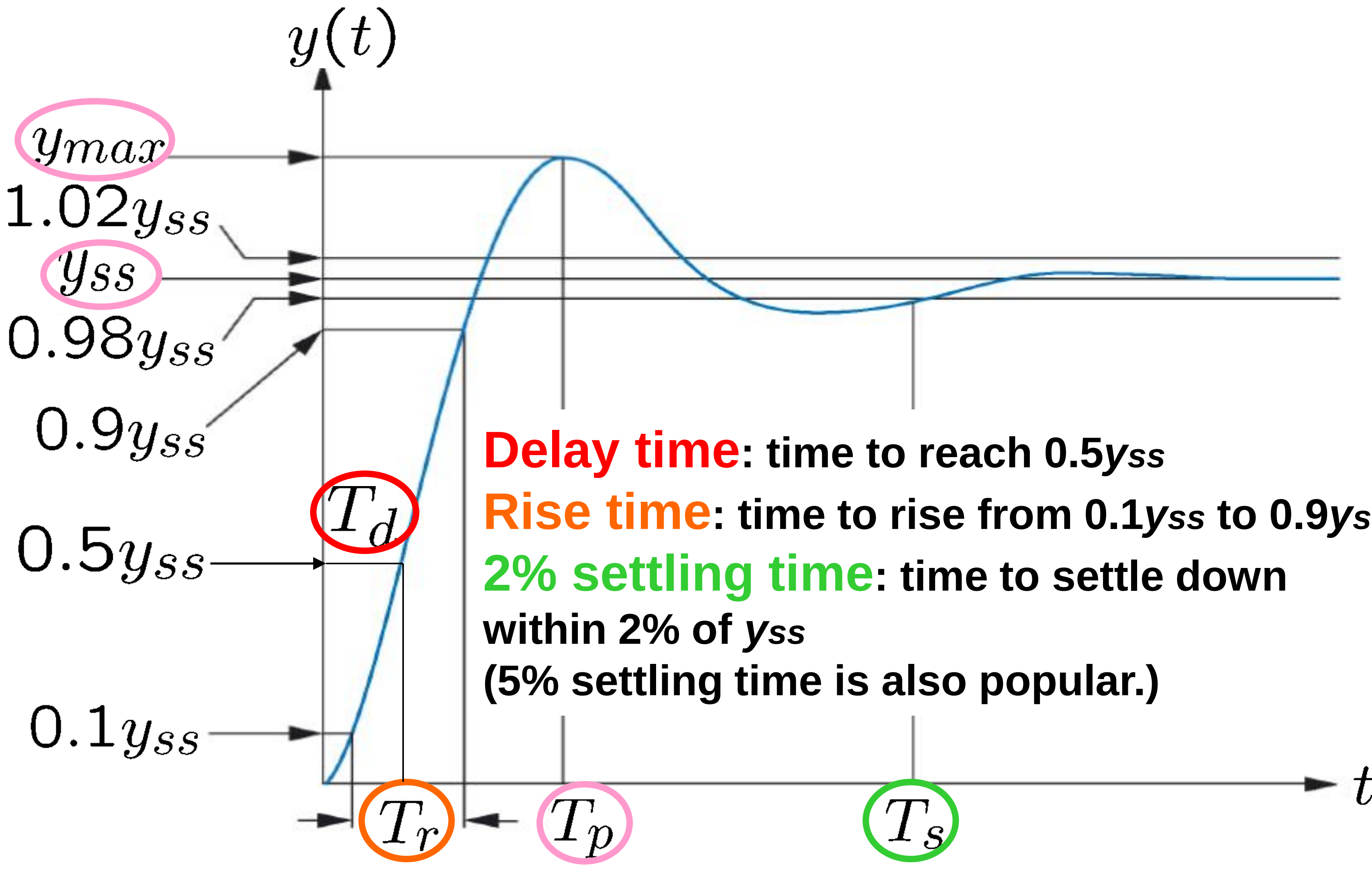


Peak value, peak time percent overshoot



$$\text{percent overshoot} = \frac{y_{max} - y_{ss}}{y_{ss}} \times 100(\%)$$

Delay, rise, and settling times



Delay time: time to reach 0.5y_{ss}

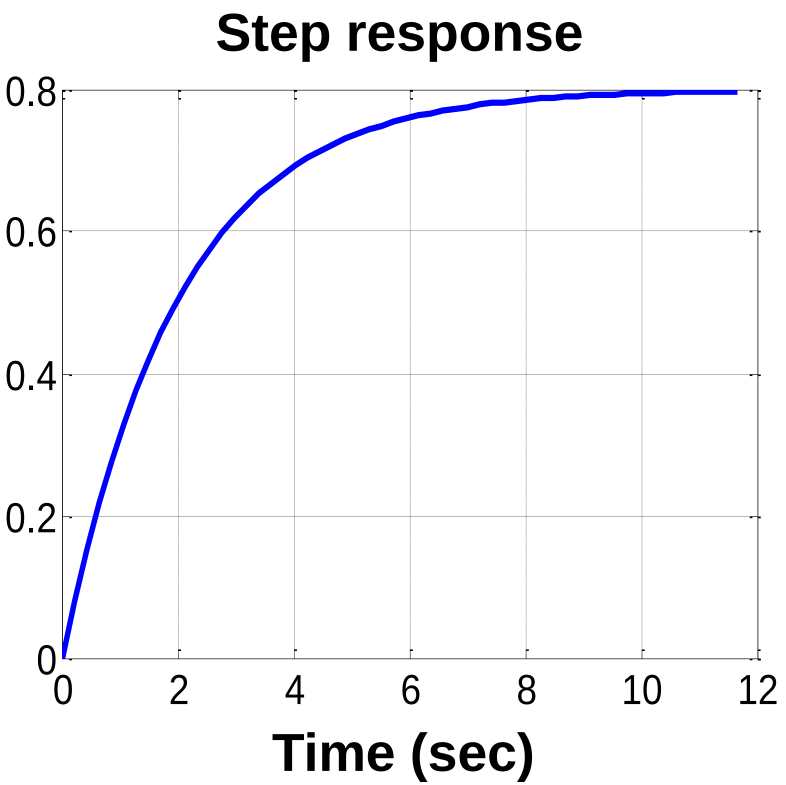
Rise time: time to rise from 0.1y_{ss} to 0.9y_{ss}

2% settling time: time to settle down within 2% of y_{ss}

(5% settling time is also popular.)

An example revisited

- For the example in Slide 6:
  - Steady-state error : 1-0.8=0.2
  - Delay time around 1.5 sec
  - Rise time around 5 sec
  - Settling time around 8 sec
- Remark: There is no peak in this case, so the following are undefined.
  - peak value
  - peak time
  - percent overshoot



Remarks on time responses

- Speed of response is measured by
  - Rise time, delay time, peak time and settling time
- Relative stability is measured by
  - Percent overshoot
- Typically ….
  - Fast response (short rise time, short peak time)

  → Large percent overshoot → Small stability margin
- In controller design, we need to take trade-off between response speed and stability.

(“***No-free-lunch theorem***” in Control Engineering)

Performance measures

- Transient response
  - Peak value
  - Peak time
  - Percent overshoot
  - Delay time
  - Rise time
  - Settling time
- Steady state response
  - Steady state error

(Next lecture)

Over the next few lectures we will connect these measures with the s-domain.

(a later lecture)

Summary

- A stable system time response consists of transient and steady state components. This is also termed natural and forced responses, respectively.
- Typical test input signals: step, ramp and parabola
- Typical response specifications: e.g. percent overshoot, settling time

- Next lecture, transient response (1^{st} and 2^{nd} order systems)